1. Stress and Strain

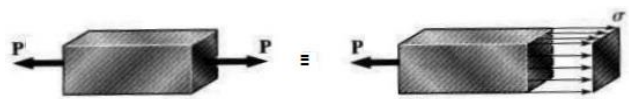
Theory at a Glance (for IES, GATE, PSU)

1.1 Stress (σ)

When a material is subjected to an external force, a resisting force is set up within the component. The internal resistance force per unit area acting on a material or intensity of the forces distributed over a given section is called the stress at a point.

- It uses original cross section area of the specimen and also known as engineering stress or conventional stress.

Therefore, $\sigma = \frac{P}{A}$



- P *is* expressed in *Newton* (N) and *A,* original area, in square meters (m^2), the stress σ will be expresses in N/ m^2. This unit is called *Pascal (Pa).*
- As *Pascal* is a small quantity, in practice, multiples of this unit is used.

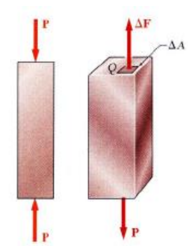
1 kPa = 10^3 Pa = 10^3 N/ m^2	(kPa = Kilo Pascal)
1 MPa = 10^6 Pa = 10^6 N/ m^2 = 1 N/mm^2	(MPa = Mega Pascal)
1 GPa = 10^9 Pa = 10^9 N/ m^2	(GPa = Giga Pascal)

Let us take an example: A rod 10 mm × 10 mm cross-section is carrying an axial tensile load 10 kN. In this rod the tensile stress developed is given by

$$(\sigma_t) = \frac{P}{A} = \frac{10\text{ kN}}{(10\text{mm} \times 10\text{mm})} = \frac{10 \times 10^3\text{ N}}{100\text{ mm}^2} = 100\text{ N/mm}^2 = 100\text{ MPa}$$

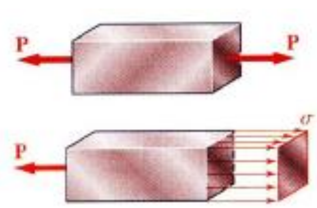
- The resultant of the internal forces for an axially loaded member is normal to a section cut perpendicular to the member axis.
- The force intensity on the shown section is defined as the normal stress.

$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \quad \text{and} \quad \sigma_{avg} = \frac{P}{A}$$

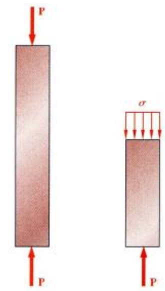


- **Tensile stress (σ_t)**

If σ > 0 the stress is tensile. i.e. The fibres of the component tend to elongate due to the external force. A member subjected to an external force tensile P and tensile stress distribution due to the force is shown in the given figure.

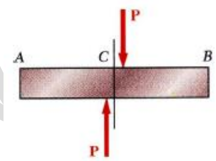


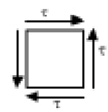
- **Compressive stress (σ_c)**



If σ < 0 the stress is compressive. i.e. The fibres of the component tend to shorten due to the external force. A member subjected to an external compressive force P and compressive stress distribution due to the force is shown in the given figure.

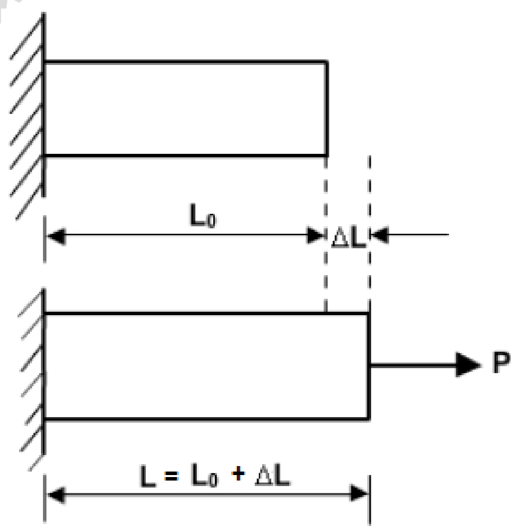
- **Shear stress (t)**





When forces are transmitted from one part of a body to other, the stresses developed in a plane parallel to the applied force are the shear stress. **Shear stress acts parallel to plane of interest. Forces *P* is applied transversely to the member *AB as shown.* The corresponding internal forces act in the plane of section *C* and are called *shearing* forces. The corresponding average shear stress** $(t) = \frac{P}{\text{Area}}$

1.2 Strain (ε)



The displacement per unit length ***(dimensionless)*** is known as strain.

- **Tensile strain (ε_t)**

The elongation per unit length as shown in the figure is known as tensile strain.

$$\varepsilon_t = \Delta L / L_o$$

It is engineering strain or conventional strain.

Here we divide the elongation to original length not actual length (L_o + D L)

Let us take an example: A rod 100 mm in original length. When we apply an axial tensile load 10 kN the final length of the rod after application of the load is 100.1 mm. So in this rod tensile strain is developed and is given by

$$(e_t) = \frac{DL}{L_o} = \frac{L - L_o}{L_o} = \frac{100.1\,\text{mm} - 100\,\text{mm}}{100\,\text{mm}} = \frac{0.1\,\text{mm}}{100\,\text{mm}} = 0.001 \text{ (Dimensionless) Tensile}$$

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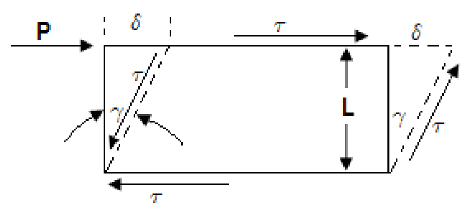
- **Compressive strain (ε_c)**

  If the applied force is compressive then the reduction of length per unit length is known as compressive strain. It is negative. Then $\varepsilon_c = (-\Delta L)/ L_o$

Let us take an example: A rod 100 mm in original length. When we apply an axial compressive load 10 kN the final length of the rod after application of the load is 99 mm. So in this rod a compressive strain is developed and is given by

$$(e_c) = \frac{D L}{L_o} = \frac{L - L_o}{L_o} = \frac{99\,\text{mm} - 100\,\text{mm}}{100\,\text{mm}} = \frac{-1\,\text{mm}}{100\,\text{mm}} = -0.01 \text{ (Dimensionless) compressive}$$

- **Shear Strain (g):** When a force P is applied tangentially to the element shown. Its edge displaced to dotted line. Where d is the lateral displacement of the upper face of the element relative to the lower face and L is the distance between these faces. Then the shear strain is $(g) = \dfrac{d}{L}$



Let us take an example: A block 100 mm × 100 mm base and 10 mm height. When we apply a tangential force 10 kN to the upper edge it is displaced 1 mm relative to lower face.

Then the direct shear stress in the element

$$(t) = \frac{10\,\text{kN}}{100\,\text{mm}' \, 100\,\text{mm}} = \frac{10' \, 10^3\,\text{N}}{100\,\text{mm}' \, 100\,\text{mm}} = 1 \text{ N/mm}^2 = 1 \text{ MPa}$$

And shear strain in the element $(g) = = \dfrac{1\,\text{mm}}{10\,\text{mm}} = 0.1$ Dimensionless

1.3 True stress and True Strain

The true stress is defined as the ratio of the load to the cross section area at any instant.

$$(s_T) = \frac{\text{load}}{\text{Instantaneous area}} = s\,(1 + e)$$

Where s and e is the engineering stress and engineering strain respectively.

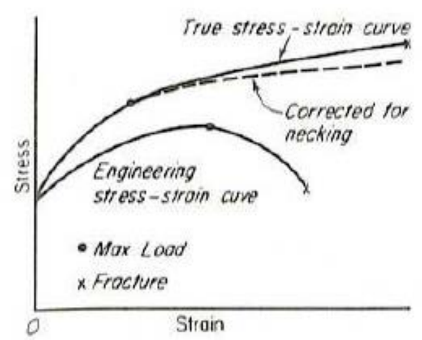
- **True strain**

$$(e_T) = \int_{L_o}^{L} \frac{dl}{l} = \ln\left(\frac{L}{L_o}\right) = \ln(1+e) = \ln\left(\frac{A_o}{A}\right) = 2\ln\left(\frac{d_o}{d}\right)$$

or engineering strain (e) $= e^{e_T} - 1$

The volume of the specimen is assumed to be constant during plastic deformation. [$\because A_o L_o = AL$] It is valid till the neck formation.

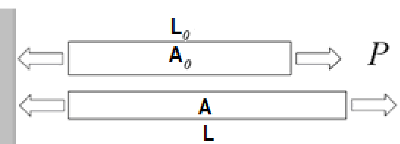
- **Comparison of engineering and the true stress-strain curves shown below:**



- The **true stress-strain curve** is also known as the **flow curve.**
- True stress-strain curve gives a true indication of deformation characteristics because it is **based on the instantaneous dimension** of the specimen.
- In engineering stress-strain curve, stress drops down after necking since it is based on the original area.
- In true stress-strain curve, the stress however increases after necking since the cross-sectional area of the specimen decreases rapidly after necking.

Let us take two examples:

(I.) Only elongation no neck formation



In the tension test of a rod shown initially it was $A_o = 50\ mm^2$ and $L_o = 100$ mm. After the application of load it's $A = 40\ mm^2$ and L = 125 mm.

Determine the true strain using changes in both length and area.

Answer: First of all we have to check that does the member forms neck or not? For that check $A_o L_o = AL$ or not?

Here 50 × 100 = 40 × 125 so no neck formation is there. Therefore true strain

*(If **no neck formation** occurs both area and gauge length can be used for a strain calculation.)*

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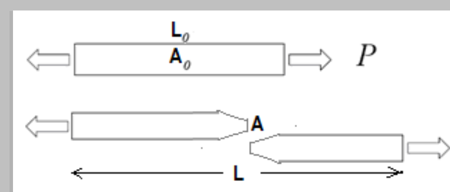
$$(\varepsilon_T) = \int_{L_o}^{L} \frac{dl}{l} = \ln\left(\frac{125}{100}\right) = 0.223$$

$$(\varepsilon_T) = \ln\left(\frac{A_o}{A}\right) = \ln\left(\frac{50}{40}\right) = 0.223$$

(II.) Elongation with neck formation



A ductile material is tested such and necking occurs then the final gauge length is L = 140 mm and the final minimum cross sectional area is A = 35 mm². Though the rod shown initially it was A_o = 50 mm² and L_o = 100 mm. Determine the true strain using changes in both length and area.

Answer: First of all we have to check that does the member forms neck or not? For that check $A_oL_o = AL$ or not?

Here $A_oL_o = 50 \times 100 = 5000$ mm³ and $AL = 35 \times 140 = 4200$ mm³. So neck formation is there. Note here $A_oL_o > AL$.

Therefore true strain

$$(\varepsilon_T) = \ln\left(\frac{A_o}{A}\right) = \ln\left(\frac{50}{35}\right) = 0.357$$

But not $(\varepsilon_T) = \int_{L_o}^{L} \frac{dl}{l} = \ln\left(\frac{140}{100}\right) = 0.336$ (it is wrong)

(***After necking,*** *gauge length gives error but area and diameter can be used for the calculation of true strain at fracture and before fracture also.*)

1.4 Hook's law

According to Hook's law the stress is directly proportional to strain i.e. normal stress (σ) $\propto$ normal strain (ε) and shearing stress (τ) $\propto$ shearing strain (γ).

$$\sigma = E\varepsilon \text{ and } \tau = G\gamma$$

The co-efficient E is called the ***modulus of elasticity*** *i.e. its resistance to elastic strain.* The co-efficient G is called the ***shear modulus of elasticity*** *or* ***modulus of rigidity.***

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1.5 Volumetric strain (ε_v)

A relationship similar to that for length changes holds for three-dimensional (volume) change. For *volumetric strain* (e_v), the relationship is $(e_v) = (V-V_0)/V_0$ or $(e_v) = \Delta V/V_0 = \frac{P}{K}$

- Where V is the final volume, V_0 is the original volume, and ΔV is the volume change.
- Volumetric strain is a ratio of values with the same units, so it also is a dimensionless quantity.
- $\Delta V/V$= *volumetric strain* = $\varepsilon_x + \varepsilon_y + \varepsilon_z = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$
- **Dilation:** The hydrostatic component of the total stress contributes to deformation by changing the area (or volume, in three dimensions) of an object. Area or volume change is called **dilation** and is positive or negative, as the volume increases or decreases, respectively. $e = \frac{p}{K}$ where p is pressure.

1.6 Young's modulus or Modulus of elasticity (E) = $\frac{PL}{A\delta} = \frac{\sigma}{\varepsilon}$

1.7 Modulus of rigidity or Shear modulus of elasticity (G) = $\frac{\tau}{\gamma} = \frac{TL}{J\theta}$

1.8 Bulk Modulus or Volume modulus of elasticity (K) = $-\frac{\Delta p}{\frac{\Delta v}{v}} = \frac{\Delta p}{\frac{\Delta R}{R}}$

1.10 Relationship between the elastic constants E, G, K, μ

$$E = 2G(1+\mu) = 3K(1-2\mu) = \frac{9KG}{3K+G}$$ **[VIMP]**

Where K = Bulk Modulus, μ= Poisson's Ratio, E= Young's modulus, G= Modulus of rigidity

- For a linearly elastic, isotropic and homogeneous material, the number of elastic constants required to relate stress and strain is two. i.e. any two of the four must be known.
- If the material is non-isotropic (i.e. anisotropic), then the elastic moduli will vary with additional stresses appearing since there is a coupling between shear stresses and normal stresses for an anisotropic material.

Let us take an example: The modulus of elasticity and rigidity of a material are 200 GPa and 80 GPa, respectively. Find all other elastic modulus.

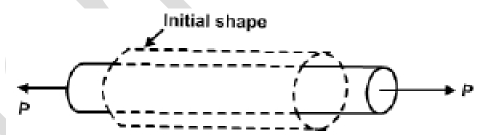
Answer: Using the relation $E = 2G(1+\mu) = 3K(1-2\mu) = \dfrac{9KG}{3K+G}$ we may find all other elastic modulus easily

Poisson's Ratio (μ) : $1+\mu = \dfrac{E}{2G}$ $\Rightarrow \mu = \dfrac{E}{2G} - 1 = \dfrac{200}{2\times 80} - 1 = 0.25$

Bulk Modulus (K) : $3K = \dfrac{E}{1-2\mu}$ $\Rightarrow K = \dfrac{E}{3(1-2\mu)} = \dfrac{200}{3(1-2\times 0.25)} = 133.33\,\text{GPa}$

1.11 Poisson's Ratio (μ)

$$= \frac{\text{Transverse strain or lateral strain}}{\text{Longitudinal strain}} = \frac{-\varepsilon_y}{\varepsilon_x}$$



(Under unidirectional stress in *x*-direction)

- The theory of isotropic elasticity allows Poisson's ratios in the range from –1 to 1/2.
- Poisson's ratio in various materials

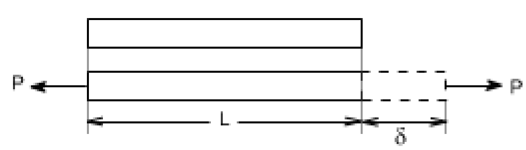
Material	Poisson's ratio	Material	Poisson's ratio
Steel	0.25 – 0.33	Rubber	0.48 – 0.5
C.I	0.23 – 0.27	Cork	Nearly zero
Concrete	0.2	Novel foam	negative

- We use cork in a bottle as the cork easily inserted and removed, yet it also withstand the pressure from within the bottle. Cork with a Poisson's ratio of nearly zero, is ideal in this application.

1.12 Elongation

- **A prismatic bar loaded in tension by an axial force P**

For a prismatic bar loaded in tension by an axial force P. The elongation of the bar can be determined as



$$\delta = \frac{PL}{AE}$$

Let us take an example: A Mild Steel wire 5 mm in diameter and 1 m long. If the wire is subjected to an axial tensile load 10 kN find its extension of the rod. (E = 200 GPa)

Answer: We know that $(\delta) = \dfrac{PL}{AE}$

Here given, Force (P) $= 10\,kN = 10 \times 1000\,\text{N}$

Length (L) = 1 m

$$\text{Area (A)} = \frac{\pi d^2}{4} = \frac{\pi \times (0.005)^2}{4}\,\text{m}^2 = 1.963 \times 10^{-5}\,\text{m}^2$$

Modulous of Elasticity $(E) = 200\,GPa = 200 \times 10^9\,\text{N/m}^2$

$$\text{Therefore Elongation } (\delta) = \frac{PL}{AE} = \frac{(10 \times 1000) \times 1}{(1.963 \times 10^{-5}) \times (200 \times 10^9)}\,\text{m} = 2.55 \times 10^{-3}\,\text{m} = 2.55\text{ mm}$$

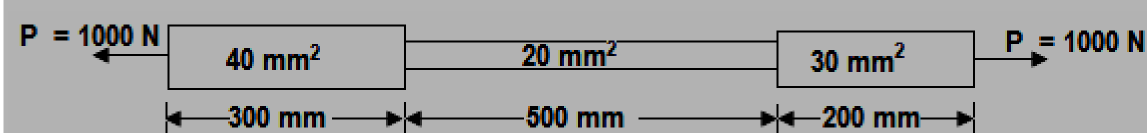
- ***Elongation of composite body***

Elongation of a bar of varying cross section $A_1, A_2, \ldots, A_n$ of lengths $l_1, l_2, \ldots, l_n$ respectively.

$$\delta = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3} - - - - - - - - + \frac{l_n}{A_n}\right]$$

Let us take an example: A composite rod is 1000 mm long, its two ends are 40 mm² and 30 mm² in area and length are 300 mm and 200 mm respectively. The middle portion of the rod is 20 mm² in area and 500 mm long. If the rod is subjected to an axial tensile load of 1000 N, find its total elongation. (E = 200 GPa).

Answer: Consider the following figure



Given, Load (P) =1000 N

Area; $(A_1) = 40\text{ mm}^2$, $A_2 = 20\text{ mm}^2$, $A_3 = 30\text{ mm}^2$

Length; $(l_1) = 300$ mm, $l_2 = 500$ mm, $l_3 = 200$ mm

$E = 200\text{ GPa} = 200 \times 10^9\text{ N/m}^2 = 200 \times 10^3\text{ N/mm}^2$

Therefore Total extension of the rod

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$$\delta = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right] = \frac{1000\,N}{200 \times 10^3\ \text{N/mm}^2} \times \left[\frac{300\,\text{mm}}{40\,\text{mm}^2} + \frac{500\,\text{mm}}{20\,\text{mm}^2} + \frac{200\,\text{mm}}{30\,\text{mm}^2}\right] = 0.196\,\text{mm}$$

- ***Elongation of a tapered body***

Elongation of a tapering rod of length 'L' due to load 'P' at the end

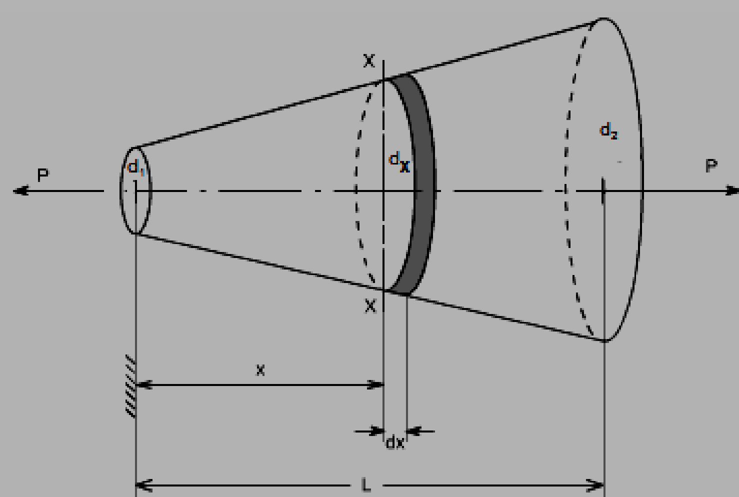
$$\delta = \frac{4PL}{\pi E d_1 d_2}$$ (d_1 and d_2 are the diameters of smaller & larger ends)

You may remember this in this way, $\delta = \frac{PL}{E\left(\frac{\pi}{4} d_1 d_2\right)}$ *i.e.* $\frac{PL}{EA_{eq}}$

Let us take an example: A round bar, of length L, tapers uniformly from small diameter d_1 at one end to bigger diameter d_2 at the other end. Show that the extension produced by a tensile axial load P is $(\delta) = \frac{4PL}{\pi d_1 d_2 E}$.

If $d_2 = 2d_1$, compare this extension with that of a uniform cylindrical bar having a diameter equal to the mean diameter of the tapered bar.

Answer: Consider the figure below d_1 be the radius at the smaller end. Then at a X cross section XX located at a distance × from the smaller end, the value of diameter 'd_x' is equal to



$$\frac{d_x}{2} = \frac{d_1}{2} + \frac{x}{L}\left(\frac{d_2}{2} - \frac{d_1}{2}\right)$$

$$\text{or} \quad d_x = d_1 + \frac{x}{L}(d_2 - d_1) = d_1(1 + kx) \qquad \text{where } k = \frac{d_2 - d_1}{L} \times \frac{1}{d_1}$$

We now taking a small strip of diameter 'd_x'and length 'd_x'at section XX.
Elongation of this section 'd_x' length

$$d(\delta) = \frac{PL}{AE} = \frac{P.dx}{\left(\frac{\pi d_x^2}{4}\right) \times E} = \frac{4P.dx}{\pi.\{d_1(1 + kx)\}^2 E}$$

Therefore total elongation of the taper bar

$$\delta = \int d(\delta) = \int_{x=0}^{x=L} \frac{4P\,dx}{\pi E d_1^2 (1 + kx)^2} = \frac{4PL}{\pi E d_1 d_2}$$

Comparison: Case-I: Where $d_2 = 2d_1$

$$\text{Elongation } (\delta_I) = \frac{4PL}{\pi E d_1 \times 2d_1} = \frac{2PL}{\pi E d_1^2}$$

Case –II: Where we use Mean diameter

$$d_m = \frac{d_1 + d_2}{2} = \frac{d_1 + 2d_1}{2} = \frac{3}{2}d_1$$

$$\text{Elongation of such bar } (\delta_{II}) = \frac{PL}{AE} = \frac{P.L}{\frac{\pi}{4}\left(\frac{3}{2}d_1\right)^2 .E} = \frac{16PL}{9\pi E d_1^2}$$

$$\frac{\text{Extension of taper bar}}{\text{Extension of uniform bar}} = \frac{2}{\frac{16}{9}} = \frac{9}{8}$$

- ***Elongation of a body due to its self weight***

(i) Elongation of a uniform rod of length 'L' due to its own weight 'W'

$$\delta = \frac{WL}{2AE}$$

The deformation of a bar under its own weight as compared to that when subjected to a direct axial load equal to its own weight ***will be half***.

(ii) Total extension produced in rod of length 'L' due to its own weight 'w' per with

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length $d = \frac{wL^2}{2EA}$

(iii) Elongation of a conical bar due to its self weight

$$d = \frac{r\,gL^2}{6E} = \frac{WL}{2A_{max}E}$$

1.13 Thermal or Temperature stress and strain

- When a material undergoes a change in temperature, it either elongates or contracts depending upon whether temperature is increased or decreased of the material.
- If the elongation or contraction is *not restricted*, i. e. *free* then the material does not experience *any stress despite the fact that it undergoes a strain.*
- The strain due to temperature change is called *thermal strain* and is expressed as,

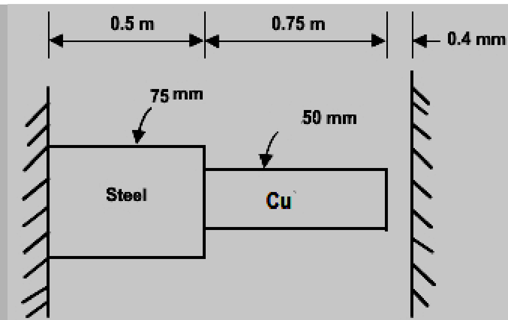
$$\varepsilon = \alpha\left(\Delta T\right)$$

- Where α is co-efficient of thermal expansion, a material property, and ΔT is the change in temperature.
- The free expansion or contraction of materials, when restrained induces stress in the material and it is referred to as *thermal stress*.

$$\sigma_t = \alpha E\left(\Delta T\right)$$ Where, E = Modulus of elasticity

- Thermal stress produces the same effect in the material similar to that of mechanical stress. A compressive stress will produce in the material with increase in temperature and the stress developed is tensile stress with decrease in temperature.

Let us take an example: A rod consists of two parts that are made of steel and copper as shown in figure below. The elastic modulus and coefficient of thermal expansion for steel are 200 GPa and 11.7 × 10^{-6} per °C respectively and for copper 70 GPa and 21.6 × 10^{-6} per °C respectively. If the temperature of the rod is raised by 50°C, determine the forces and stresses acting on the rod.



Answer: If we allow this rod to freely expand then free expansion

$$\delta_T = \alpha (\Delta T) L$$
$$= (11.7 \times 10^{-6}) \times 50 \times 500 + (21.6 \times 10^{-6}) \times 50 \times 750$$
$$= 1.1025 \text{ mm (Compressive)}$$

But according to diagram only free expansion is 0.4 mm.

Therefore restrained deflection of rod =1.1025 mm – 0.4 mm = 0.7025 mm

Let us assume the force required to make their elongation vanish be P which is the reaction force at the ends.

$$\delta = \left(\frac{PL}{AE}\right)_{Steel} + \left(\frac{PL}{AE}\right)_{Cu}$$

$$\text{or} \quad 0.7025 = \frac{P \times 500}{\left\{\frac{\pi}{4} \times (0.075)^2\right\} \times (200 \times 10^9)} + \frac{P \times 750}{\left\{\frac{\pi}{4} \times (0.050)^2\right\} \times (70 \times 10^9)}$$

$$\text{or } P = 116.6 \text{ kN}$$

Therefore, compressive stress on steel rod

$$\sigma_{Steel} = \frac{P}{A_{Steel}} = \frac{116.6 \times 10^3}{\frac{\pi}{4} \times (0.075)^2} \text{N/m}^2 = 26.39 \text{ MPa}$$

And compressive stress on copper rod

$$\sigma_{Cu} = \frac{P}{A_{Cu}} = \frac{116.6 \times 10^3}{\frac{\pi}{4} \times (0.050)^2} \text{N/m}^2 = 59.38 \text{ MPa}$$

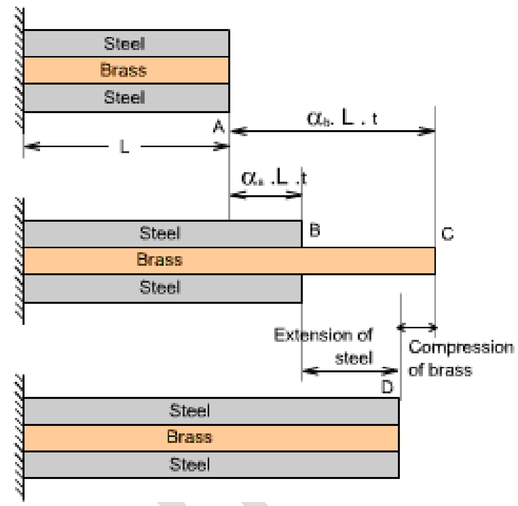
1.14 Thermal stress on Brass and Mild steel combination

A brass rod placed within a steel tube of exactly same length. The assembly is making in such a way that elongation of the combination will be same. To calculate the stress induced in the brass rod, steel tube when the combination is raised by t°C then the following analogy have to do.

(a) Original bar before heating.

(b) Expanded position if the members are allowed to expand freely and independently after heating.

(c) Expanded position of the compound bar i.e. final position after heating.



- Compatibility Equation:

$$d = d_{st} + d_{sf} = d_{Bt} - d_{Bf}$$

- Equilibrium Equation:

$$s_s A_s = s_B A_B$$

Assumption:

1. $L = L_s = L_B$
2. $a_b > a_s$
3. Steel - Tension
   Brass - Compression

Where, δ = Expansion of the compound bar = AD in the above figure.

d_{st} = Free expansion of the steel tube due to temperature rise t°C = $\alpha_s L t$
= AB in the above figure.

d_{sf} = Expansion of the steel tube due to internal force developed by the unequal expansion.
= BD in the above figure.

d_{Bt} = Free expansion of the brass rod due to temperature rise t°C = $a_b L t$
= AC in the above figure.

d_{Bf} = Compression of the brass rod due to internal force developed by the unequal expansion.
= BD in the above figure.

And in the equilibrium equation

Tensile force in the steel tube = Compressive force in the brass rod

Where, σ_s = Tensile stress developed in the steel tube.

σ_B = Compressive stress developed in the brass rod.

A_s = Cross section area of the steel tube.

A_B = Cross section area of the brass rod.

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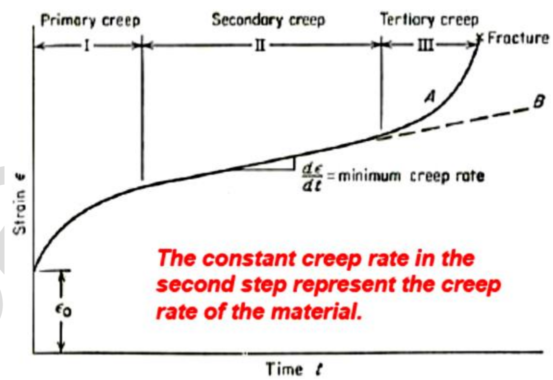
Let us take an example: See the Conventional Question Answer section of this chapter and the question is "***Conventional Question IES-2008***" and it's answer.

1.15 Creep

When a member is subjected to a constant load over a long period of time it undergoes a slow permanent deformation and this is termed as "creep". This is dependent on temperature. Usually at elevated temperatures creep is high.

- The materials have its own different melting point; each will creep when the homologous temperature > 0.5. Homologous temp $= \dfrac{\text{Testing temperature}}{\text{Melting temperature}} > 0.5$

A typical creep curve shows three distinct stages with different creep rates. After an initial rapid elongation ε_o, the creep rate decrease with time until reaching the steady state.



1) ***Primary creep*** is a period of transient creep. The creep resistance of the material increases due to material deformation.

2) ***Secondary creep*** provides a nearly constant creep rate. The average value of the creep rate during this period is called the minimum creep rate. A stage of balance between competing.
   Strain hardening and ***recovery*** (softening) of the material.

3) ***Tertiary creep*** shows a rapid increase in the creep rate due to effectively reduced cross-sectional area of the specimen leading to *creep rupture* or failure. In this stage *intergranular* cracking and/or formation of voids and cavities occur.

1.16 If a load P is applied suddenly to a bar then the stress & strain induced will be ***double*** than those obtained by an equal load applied gradually.

Impact Stresses:

1.17 Stress produced by a load P in falling from height 'h'

$$\sigma_d = \sigma\left[1+\sqrt{1+\frac{2h}{\varepsilon L}}\right] \quad \sigma,$$

ε being stress & strain produced by static load P & L=length of bar.

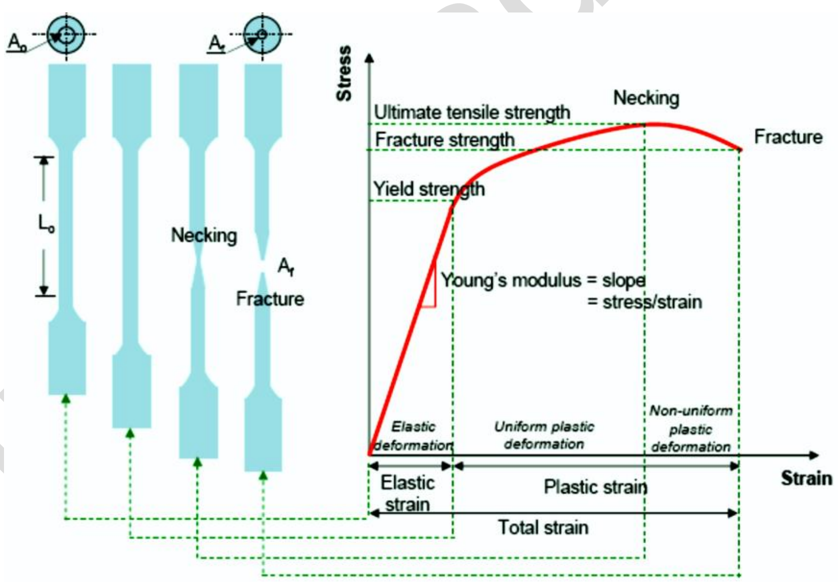
$$= \frac{A}{P}\left[1+\sqrt{1+\frac{2AEh}{PL}}\right]$$

1.18 Loads shared by the materials of a compound bar made of bars x & y due to load W,

$$P_x = W.\frac{A_xE_x}{A_xE_x + A_yE_y} \quad \text{and} \quad P_y = W.\frac{A_yE_y}{A_xE_x + A_yE_y}$$

1.19 Elongation of a compound bar, $\delta = \dfrac{PL}{A_xE_x + A_yE_y}$

1.20 Tension Test

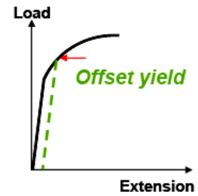


i) **True elastic limit:** based on micro-strain measurement at strains on order of 2×10^{-6}. Very low value and is related to the motion of a few hundred dislocations.

ii) **Proportional limit:** the highest stress at which stress is directly proportional to strain.

iii) **Elastic limit:** is the greatest stress the material can withstand without any measurable permanent strain after unloading. Elastic limit > proportional limit.

iv) Yield strength is the stress required to produce a small specific amount of deformation. The offset yield strength can be determined by the stress corresponding to the intersection of the stress-strain curve and a line parallel to the elastic line offset by a strain of 0.2 or 0.1%. (ε = 0.002 or 0.001).



- The offset yield stress is referred to proof stress either at 0.1 or 0.5% strain used for design and specification purposes to avoid the practical difficulties of measuring the elastic limit or proportional limit.

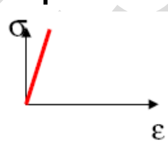
v) Tensile strength or ultimate tensile strength (UTS) σ_u is the maximum load P_{max} divided by the original cross-sectional area A_o of the specimen.

vi) % Elongation, $= \dfrac{L_f - L_o}{L_o}$, is chiefly influenced by uniform elongation, which is dependent on the strain-hardening capacity of the material.

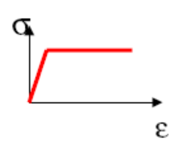
vii) Reduction of Area: $q = \dfrac{A_o - A_f}{A_o}$

- Reduction of area is more a measure of the deformation required to produce failure and its chief contribution results from the necking process.
- Because of the complicated state of stress state in the neck, values of reduction of area are dependent on specimen geometry, and deformation behaviour, and they should not be taken as true material properties.
- RA is the most structure-sensitive ductility parameter and is useful in detecting quality changes in the materials.

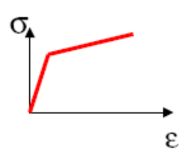
viii) Stress-strain response



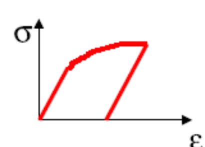
Linear elastic



Linear elastic-perfectly plastic



Linear elastic-hardening plastic



Linear elastic-hardening plasticity with unloading

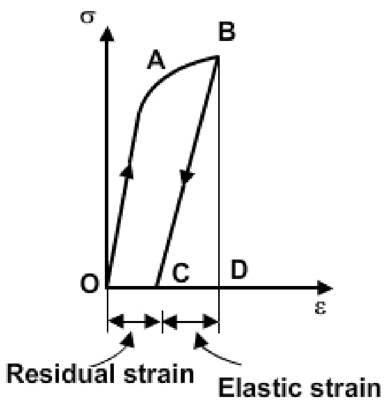
1.21 Elastic strain and Plastic strain

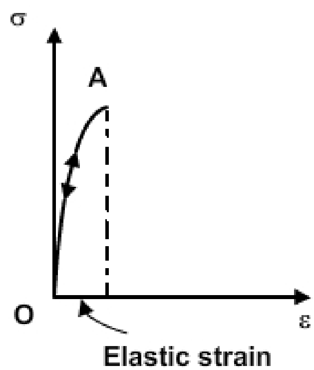
The strain present in the material after unloading is called the **residual strain or plastic strain** and the strain disappears during unloading is termed as **recoverable or elastic strain.**

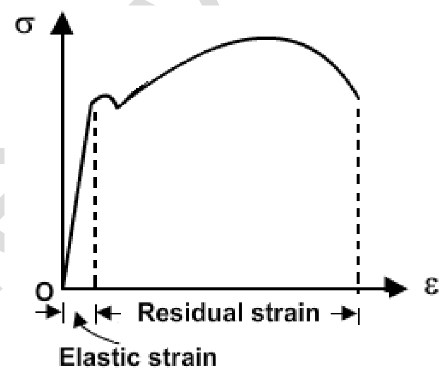
Equation of the straight line CB is given by

$$\sigma = \epsilon_{total} \times E - \epsilon_{Plastic} \times E = \epsilon_{Elastic} \times E$$

Carefully observe the following figures and understand which one is Elastic strain and which one is Plastic strain







Let us take an example: A 10 mm diameter tensile specimen has a 50 mm gauge length. The load corresponding to the 0.2% offset is 55 kN and the maximum load is 70 kN. Fracture occurs at 60 kN. The diameter after fracture is 8 mm and the gauge length at fracture is 65 mm. Calculate the following properties of the material from the tension test.

(i) % Elongation

(ii) Reduction of Area (RA) %

(iii) Tensile strength or ultimate tensile strength (UTS)

(iv) Yield strength

(v) Fracture strength

(vi) If E = 200 GPa, the elastic recoverable strain at maximum load

(vii) If the elongation at maximum load (the uniform elongation) is 20%, what is the plastic strain at maximum load?

Answer: Given, Original area $\left(A_0\right) = \frac{\pi}{4}\times\left(0.010\right)^2 \text{ m}^2 = 7.854\times10^{-5}\text{m}^2$

Area at fracture $\left(A_f\right) = \frac{\pi}{4}\times\left(0.008\right)^2 \text{ m}^2 = 5.027\times10^{-5}\text{m}^2$

Original gauge length (L_0) = 50 mm

Gauge length at fracture (L) = 65 mm

Therefore

(i) % Elongation $= \frac{L - L_0}{L_0}\times100\% = \frac{65-50}{50}\times100 = 30\%$

(ii) Reduction of area (RA) = $q = \frac{A_0 - A_f}{A_0}\times100\% = \frac{7.854-5.027}{7.854}\times100\% = 36\%$

(iii) Tensile strength or Ultimate tensile strength (UTS), $\sigma_u = \frac{P_{max}}{A_o} = \frac{70\times10^3}{7.854\times10^{-5}}\text{N/m}^2 = 891 \text{ MPa}$

(iv) Yield strength $\left(\sigma_y\right) = \frac{P_y}{A_o} = \frac{55\times10^3}{7.854\times10^{-5}}\text{N/m}^2 = 700 \text{ MPa}$

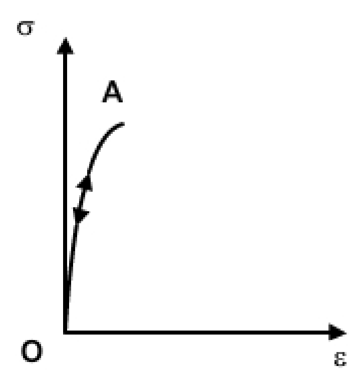
(v) Fracture strength $\left(\sigma_F\right) = \frac{P_{Fracture}}{A_o} = \frac{60\times10^3}{7.854\times10^{-5}}\text{N/m}^2 = 764\,\text{MPa}$

(vi) Elastic recoverable strain at maximum load $\left(\varepsilon_E\right) = \frac{P_{\max}/A_o}{E} = \frac{891\times10^6}{200\times10^9} = 0.0045$

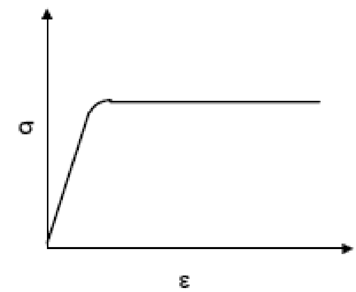
(vii) Plastic strain $\left(\varepsilon_P\right) = \varepsilon_{total} - \varepsilon_E = 0.2000 - 0.0045 = 0.1955$

1.22 Elasticity

This is the property of a material to regain its original shape after deformation when the external forces are removed. When the material is in elastic region the strain disappears completely after removal of the load, The stress-strain relationship in elastic region need not be linear and can be non-linear (example rubber). The maximum stress value below which the strain is fully recoverable is called the *elastic limit*. It is represented by point A in figure. All materials are elastic to some extent but the degree varies, for example, both mild steel and rubber are elastic materials but steel is more elastic than rubber.

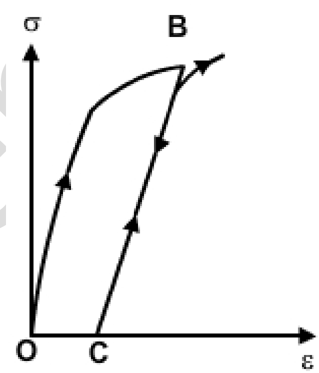


1.23 Plasticity



When the stress in the material exceeds the elastic limit, the material enters into plastic phase where the strain can no longer be completely removed. Under plastic conditions materials ideally deform without any increase in stress. A typical stress strain diagram for an elastic-perfectly plastic material is shown in the figure. Mises-Henky criterion gives a good starting point for plasticity analysis.

1.24 Strain hardening



If the material is reloaded from point C, it will follow the previous unloading path and line CB becomes its new elastic region with elastic limit defined by point B. Though the new elastic region CB resembles that of the initial elastic region OA, the internal structure of the material in the new state has changed. The change in the microstructure of the material is clear from the fact that the ductility of the material has come down due to strain hardening. When the material is reloaded, it follows the same path as that of a virgin material and fails on reaching the ultimate strength which remains unaltered due to the intermediate loading and unloading process.

1.25 Stress reversal and stress-strain hysteresis loop

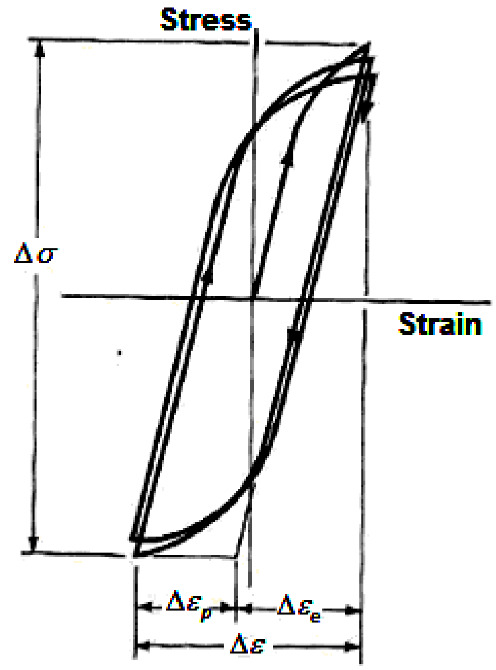
We know that fatigue failure begins at a local discontinuity and when the stress at the discontinuity exceeds elastic limit there is plastic strain. The cyclic plastic strain results crack propagation and fracture.

When we plot the experimental data with reversed loading and the true stress strain hysteresis loops is found as shown figure.

Due to cyclic strain the elastic limit increases for annealed steel and decreases for cold drawn steel.

Here the stress range is $\Delta\sigma$. $\Delta\varepsilon_p$ and $\Delta\varepsilon_e$ are the plastic and elastic strain ranges, the total strain range being $\Delta\varepsilon$. Considering that the total strain amplitude can be given as

$$\Delta\varepsilon = \Delta\varepsilon_p + \Delta\varepsilon_e$$



True stress-strain plot with a number of stress reversals

OBJECTIVE QUESTIONS (GATE & IES)

Previous 10-Years GATE Questions

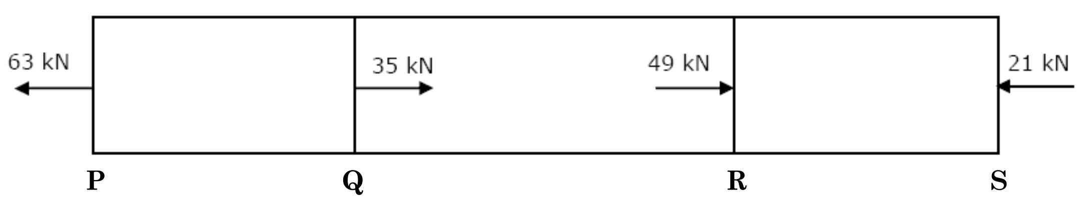
GATE-1. A steel bar of 40 mm × 40 mm square cross-section is subjected to an axial compressive load of 200 kN. If the length of the bar is 2 m and E = 200 GPa, the elongation of the bar will be: [GATE-2006]
(a) 1.25 mm (b) 2.70 mm (c) 4.05 mm (d) 5.40 mm

GATE-2. The ultimate tensile strength of a material is 400 MPa and the elongation up to maximum load is 35%. If the material obeys power law of hardening, then the true stress-true strain relation (stress in MPa) in the plastic deformation range is: [GATE-2006]
(a) $\sigma = 540\varepsilon^{0.30}$ (b) $\sigma = 775\varepsilon^{0.30}$ (c) $\sigma = 540\varepsilon^{0.35}$ (d) $\sigma = 775\varepsilon^{0.35}$

GATE-3. An axial residual compressive stress due to a manufacturing process is present on the outer surface of a rotating shaft subjected to bending. Under a given bending load, the fatigue life of the shaft in the presence of the residual compressive stress is: [GATE-2008]
(a) Decreased
(b) Increased or decreased, depending on the external bending load
(c) Neither decreased nor increased (d) Increased

GATE-6. A rod of length *L* and diameter *D* is subjected to a tensile load *P*. Which of the following is sufficient to calculate the resulting change in diameter?
(a) Young's modulus (b) Shear modulus **[GATE-2008]**
(c) Poisson's ratio (d) Both Young's modulus and shear modulus

GATE-10. A bar having a cross-sectional area of 700mm² is subjected to axial loads at the positions indicated. The value of stress in the segment QR is: [GATE-2006]



(a) 40 MPa (b) 50 MPa (c) 70 MPa (d) 120 MPa

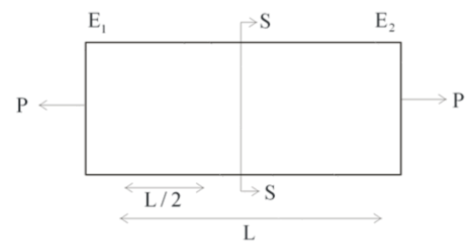
GATE-14. A rod of length L having uniform cross-section area A is subjected to a tensile force

D as shown in fig. If the young's modulus of the material varies linearly from E_1 to

E_2 along the length of the rod, the normal stress developed at the section 5.5 is **[GATE:2013]**

(a) $\frac{P}{A}$ (b) $\frac{P\left(E_1 - E_2\right)}{A\left(E_1 + E_2\right)}$

(c) $\frac{PE_2}{AE_1}$ (d) $\frac{PE_1}{AE_2}$



GATE-15. A solid steel cube constrained on all six fucos is heated so that the temperature rises uniformly by ΔT. If the thermal coefficient of the material is $\propto$. Young's modules is E & the Poison's ratio is V, the thermal stress developed in the cube due to heating is **[GATE:2012]**

(a) $\frac{\alpha \Delta TE}{\left(1-2\upsilon\right)}$ (b) $\frac{-2 \propto (\Delta T)E}{\left(1-2\upsilon\right)}$

(c) $\frac{-3 \propto \Delta TE}{\left(1-2\upsilon\right)}$ (d) $\frac{\propto \Delta TE}{3\left(1-2\upsilon\right)}$

Previous 10-Years IES Questions

IES-2. Which one of the following statements is correct? **[IES 2007]**
A beam is said to be of uniform strength, if

(a) The bending moment is the same throughout the beam
(b) The shear stress is the same throughout the beam
(c) The deflection is the same throughout the beam
(d) The bending stress is the same at every section along its longitudinal axis

IES-3. Which one of the following statements is correct? **[IES-2006]**
Beams of uniform strength vary in section such that

(a) Bending moment remains constant (b) Deflection remains constant
(c) Maximum bending stress remains constant (d) Shear force remains constant

IES-4. Two tapering bars of the same material are subjected to a tensile load P. The lengths of both the bars are the same. The larger diameter of each of the bars is D. The diameter of the bar A at its smaller end is D/2 and that of the bar B is D/3. What is the ratio of elongation of the bar A to that of the bar B? [IES-2006]

(a) 3 : 2 (b) 2: 3 (c) 4 : 9 (d) 1: 3

IES-7. If a piece of material neither expands nor contracts in volume when subjected to stresses, then the Poisson's ratio must be: [IES-2011]

(a) Zero (b) 0.25 (c) 0.33 (d) 0.5

IES-8. What is the phenomenon of progressive extension of the material i.e., strain increasing with the time at a constant load, called? [IES 2007]

(a) Plasticity (b) Yielding (c) Creeping (d) Breaking

IES-9. E, G, K and μ represent the elastic modulus, shear modulus, bulk modulus and Poisson's ratio respectively of a linearly elastic, isotropic and homogeneous material. To express the stress-strain relations completely for this material, at least [IES-2006]

(a) E, G and μ must be known (b) E, K and μ must be known
(c) Any two of the four must be known (d) All the four must be known

IES-10. What are the materials which show direction dependent properties, called?

(a) Homogeneous materials (b) Viscoelastic materials **[IES 2007]**
(c) Isotropic materials (d) Anisotropic materials

IES-11. An orthotropic material, under plane stress condition will have: [IES-2006]

(a) 15 independent elastic constants (b) 4 independent elastic constants
(c) 5 independent elastic constants (d) 9 independent elastic constants

IES-12. Young's modulus of elasticity and Poisson's ratio of a material are 1.25×10^5 MPa and 0.34 respectively. The modulus of rigidity of the material is: [IAS 1994, IES-1995, 2001, 2002, 2007]

(a) 0.4025×10^5 Mpa (b) 0.4664×10^5 Mpa
(c) 0.8375×10^5 MPa (d) 0.9469×10^5 MPa

IES-13. What is the relationship between the linear elastic properties Young's modulus (E), rigidity modulus (G) and bulk modulus (K)? [IES-2008]

(a) $\frac{1}{E} = \frac{9}{K} + \frac{3}{G}$ (b) $\frac{3}{E} = \frac{9}{K} + \frac{1}{G}$ (c) $\frac{9}{E} = \frac{3}{K} + \frac{1}{G}$ (d) $\frac{9}{E} = \frac{1}{K} + \frac{3}{G}$

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IES-14. **What is the relationship between the liner elastic properties Young's modulus (E), rigidity modulus (G) and bulk modulus (K)?** **[IES-2009]**

(a) $E = \frac{KG}{9K+G}$ (b) $E = \frac{9KG}{K+G}$ (c) $E = \frac{9KG}{K+3G}$ (d) $E = \frac{9KG}{3K+G}$

IES-16. **The modulus of elasticity for a material is 200 GN/m² and Poisson's ratio is 0.25. What is the modulus of rigidity?** **[IES-2004]**

(a) 80 GN/m² (b) 125 GN/m² (c) 250 GN/m² (d) 320 GN/m²

IES-17. **Consider the following statements:** **[IES-2009]**

1. **Two-dimensional stresses applied to a thin plate in its own plane represent the plane stress condition.**
2. **Under plane stress condition, the strain in the direction perpendicular to the plane is zero.**
3. **Normal and shear stresses may occur simultaneously on a plane.**

Which of the above statements is/are correct?

(a) 1 only (b) 1 and 2 (c) 2 and 3 (d) 1 and 3

IES-18. **Materials which show direction dependent properties are called :** **[IES-2011]**

(a) Homogeneous (b) Viscoelastic
(c) Isotropic (d) Anisotropic

IES-19. **Eight bolts are to be selected for fixing the cover plate of a cylinder subjected to a maximum load of 980·175 kN. If the design stress for the bolt material is 315 N/mm², what is the diameter of each bolt?** **[IES-2008]**

(a) 10 mm (b) 22 mm (c) 30 mm (d) 36 mm

IES-22. **Which one of the following is correct?** **[IES-2008]**
When a nut is tightened by placing a washer below it, the bolt will be subjected to:

(a) Compression only (b) Tension
(c) Shear only (d) Compression and shear

IES-23. **A 100 mm × 5 mm × 5 mm steel bar free to expand is heated from 15°C to 40°C. What shall be developed?** **[IES-2008]**

(a) Tensile stress (b) Compressive stress (c) Shear stress (d) No stress

IES-24. **Which one of the following statements is correct?** **[GATE-1995; IES 2007]**
If a material expands freely due to heating, it will develop

(a) Thermal stress (b) Tensile stress (c) Compressive stress (d) No stress

IES-25. **A steel rod 10 mm in diameter and 1m long is heated from 20°C to 120°C, E = 200 GPa and α = 12 × 10^{-6} per °C. If the rod is not free to expand, the thermal stress developed is:** **[IAS-2003, IES-1997, 2000, 2006]**
(a) 120 MPa (tensile) (b) 240 MPa (tensile)
(c) 120 MPa (compressive) (d) 240 MPa (compressive)

IES-28. **If a rod expands freely due to heating, it will develop:** **[IES-2011]**
(a) Bending stress (b) Thermal stress
(c) No stress (d) Compressive stress

IES-30. **Resilience of a material becomes important when it is subjected to:** **[IES-2011]**
(a) Fatigue (b) Thermal stresses
(c) Shock loading (d) Pure static loading

IES-31. **In a tensile test, near the elastic limit zone** **[IES-2006]**
(a) Tensile stress increases at a faster rate
(b) Tensile stress decreases at a faster rate
(c) Tensile stress increases in linear proportion to the stress
(d) Tensile stress decreases in linear proportion to the stress

IES-34. **Assertion (A):** A cast iron specimen shall fail due to shear when subjected to a compressive load.
Reason (R): Shear strength of cast iron in compression is more than half its compressive strength. **[IES-2010]**

IES-35. **Assertion (A):** A plane state of stress always results in a plane state of strain.
Reason (R): A uniaxial state of stress results in a three-dimensional state of strain.
[IES-2010]

IES-36. **Match List-I with List-II and select the correct answer using the code given below the lists:** **[IES-2011]**

List-I	List-II
A. Elasticity	**1.** Deform non elastically without fracture
B. Malleability	**2.** Undergo plastic deformation under tensile load
C. Ductility	**3.** Undergo plastic deformation under compressive load
D. Plasticity	**4.** Return to its original shape on unloading

Code :

	A	B	C	D		A	B	C	D
(a)	1	2	3	4	(b)	4	2	3	1

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(c) 1 3 2 4 (d) 4 3 2 1

IES-37. **A road of length 1 tapers uniformly form a diameters D at one end to a diameter 'd' at the other. The young's modulus of the material is E. The extension caused by an axial load P is**
[IES:2012]

(a) $\frac{\Delta Pl}{\pi\left(D^2-d^2\right)E}$ **(b)** $\frac{\Delta Pl}{\pi\left(D^2+d^2\right)E}$

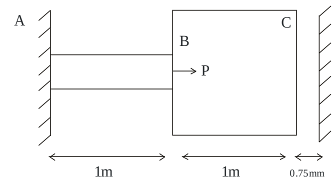
(c) $\frac{\Delta Pl}{\pi DdE}$ **(d)** $\frac{2Pl}{\pi DdE}$

IES-38. **Consider the following statements** **[IES:2013]**
1. **State of plane stress occurs at the surface**
2. **State of plane strain occurs at the surface**
3. **State of plane stress occurs in the interior part of the plate**
4. **State of plane strain occurs in the interior part of the plate**

Which of these statements are correct?

(a) 1 and 3 **(b)** 2 and 4
(c) 1 and 4 **(d)** 2 and 3

IES39. **In the arrangement as shown in the figure the stepped steel bar ABC is loaded by a load P. The material has young's modulus E=200 GPa and the two portions AB and BC have area of cross section 1cm^2 and $2\,\text{cm}^2$ respectively. The magnitude of load P required to fill up the gap of 0.75 mm is**
[IES:2013]



(a) 10KN **(b) 15KN**
(c) 20KN **(d) 25KN**

IES-40. **The modulus of rigidity and the bulk modules of a material are found as 10 GPA and 150 GPa respectively** **[IES:2014]**
Then
1. **Elasticity modulus is 200 GPa**
2. **Poison's ratio is 0.22**
3. **Elasticity modules is 182 GPa**
4. **Poison's ratio is 0.3**

Which of the above statements are correct ?

(a) 1 & 2 **(b) 1& 4**
(c) 2 & 3 **(d) 3&4**

IES-41. **A steel rod, 2m long, is held between two walls and heated form $20°\text{C}$ to $60°\text{C}$. Young's modulus and coefficient of liner expansion of the rod material are $200 \times 10^3\,\text{MPa}$ and 10×10^{-6} respectively. The stress induced in the rod, if walls field by 2mm, is** **[IES:2014]**

(a) 60 MPa tensile
(b) 80 MPa tensile
(c) 80 MPa compressive
(d) 60 MPa compressive

IES-42. **An Al bar of 8m length & a steel bar of 5m long are kept at $30°\text{C}$. If the ambient temperature is raised gradually, at what temperature the Al bar** $\left(\propto \text{st} = 12 \times 10^{-10} \,\&\, \propto \text{Al} = 23 \times 10^{-6} / °\text{C}\right)$ **[IES:2014]**

(a) $50.7°\text{C}$
(b) $69.0°\text{C}$
(c) $143.7°\text{C}$
(d) $33.7°\text{C}$

IES-43. **A copper rod of 2 cm diameter is completely encased in a steel tube of inner diameter 2 cm and outer diameter 4cm. Under an axial load, the stress in the steel tube is** $100\,N/mm^2$. **If Es=2Ec, then the stress in the copper rod is. [IES:2015]**

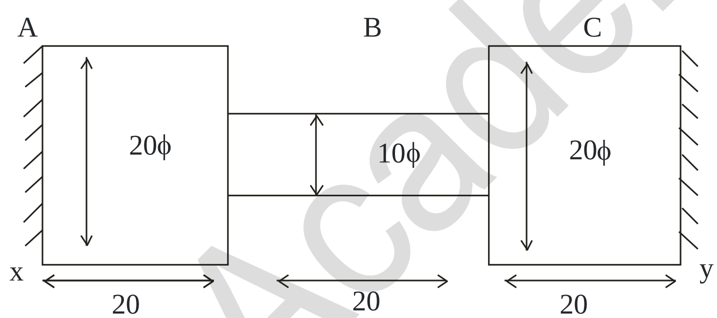
(a) $50\,N/mm^2$

(b) $33.33\,N/mm^2$

(c) $100\,N/mm^2$

(d) $300\,N/mm^2$

IES-44. **The fig shows a steel piece of diameter 20mm at A and C, and 10mm at B. The length of three section A,B & C are each equal to 20mm. The piece is held between two rigid surfaces X and Y . The coefficient of linear expansions** $\propto = 1.2\times10^{-5}/°C$ **&** $E = 2\times10^{5}\,MPa$ for steel **[IES:2015]**



When the temp of this piece increases by $50°C$ **the stresses in section A and B are**

(a) 120 MPa & 480 MPa
(b) 60 MPa & 240 MPa
(c) 120 MPa & 120 MPa
(d) 60 MPa & 120 MPa

IES-45. **For a material following hooker's law, the values of elastic & shear moduli are** $3\times10^{5}\,MPa$ & $1.2\times10^{5}\,MPa$ **suspectively. The value of bulk modulus is** **[IES: 2015]**

(a) $1.5\times10^{5}\,MPa$

(b) $2\times10^{5}\,MPa$

(c) $2.5\times10^{5}\,MPa$

(d) $3\times10^{5}\,MPa$

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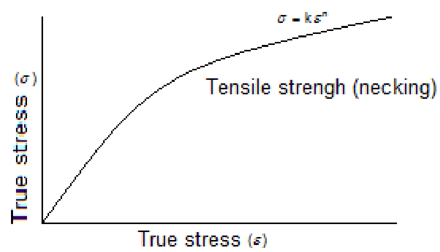
Answers with Explanation (Objective)

Previous 10-Years GATE Answers

GATE-2. Ans. (a) $dL = \frac{PL}{AE} = \frac{(200 \times 1000) \times 2}{(0.04 \times 0.04) \times 200 \times 10^9} \text{m} = 1.25 \text{mm}$

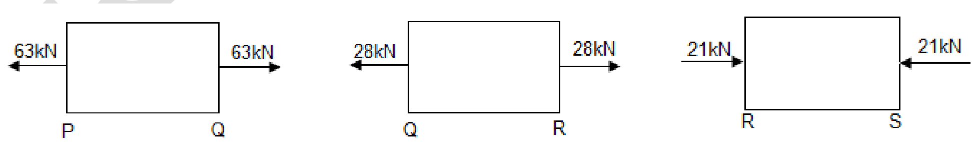
GATE-3. Ans. (c) A true stress – true strain curve in tension $\sigma = k\varepsilon^n$
k = Strength co-efficient = 400 × (1.35) = 540 MPa
n = Strain – hardening exponent = 0.35



GATE-5. Ans. (d)

GATE-6. Ans. (d) For longitudinal strain we need Young's modulus and for calculating transverse strain we need Poisson's ratio. We may calculate Poisson's ratio from $E = 2G(1 + \mu)$ for that we need Shear modulus.

GATE-10. Ans. (a)



F.B.D

$$\sigma_{QR} = \frac{P}{A} = \frac{28000}{700} \text{MPa} = 40 \text{ MPa}$$

$$F_y \times 50 = F \times 100 \quad \text{or} \quad F = \frac{F_y \times 50}{100} = \frac{10000 \times 50}{100} = 5000 \text{N}$$

GATE-14. Ans. (a)

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GATE-15. Ans. (a)

Strain in x direction,

$$= -\propto (\Delta T) = \frac{\delta x}{E} - \frac{\upsilon \delta y}{E} - \frac{\upsilon \delta z}{E}$$

$$\delta x = \delta y = \delta z = \delta$$

$$-\propto \Delta T = \frac{\delta}{E} - \frac{\upsilon\delta}{E} - \frac{\delta\upsilon}{E}$$

$$\delta = \frac{-\propto\left(\Delta T\right)E}{1-2\upsilon}$$

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Previous 10-Years IES Answers

IES-2. Ans. (d)

IES-3. Ans. (c)

IES-4. Ans. (c)

IES-4. Ans. (c) Actual elongation of the bar $(\delta l)_{act} = \dfrac{PL}{\left(\dfrac{\pi}{4} d_1 d_2\right)E} = \dfrac{PL}{\left(\dfrac{\pi}{4} \times 1.1D \times 0.9D\right)E}$

Calculated elongation of the bar $(\delta l)_{Cal} = \dfrac{PL}{\dfrac{\pi D^2}{4} \times E}$

$$\therefore \text{Error}(\%) = \frac{(\delta l)_{act} - (\delta l)_{cal}}{(\delta l)_{cal}} \times 100 = \left(\frac{D^2}{1.1D \times 0.9D} - 1\right) \times 100\% = 1\%$$

$\therefore \mu = 0.5$

IES-7. Ans. (c)

IES-8. Ans. (b)

IES-9. Ans. (c)

IES-10. Ans. (d)

IES-11. Ans. (d)

IES-12. Ans. (b) $E = 2G(1+\mu)$ or $1.25 \times 10^5 = 2G(1 + 0.34)$ or $G = 0.4664 \times 10^5$ MPa

IES-13. Ans. (d) $E = 2G(1+\mu) = 3K(1-2\mu) = \dfrac{9KG}{3K+G}$

IES-14. Ans. (d) $E = 2G(1+\mu) = 3K(1-2\mu) = \dfrac{9KG}{3K+G}$

IES-16. Ans. (a) $E = 2G(1+\mu)$ or $G = \dfrac{E}{2(1+\mu)} = \dfrac{200}{2 \times (1+0.25)} = 80\ \text{GN/m}^2$

IES-17. Ans. (d) Under plane stress condition, the strain in the direction perpendicular to the plane is not zero. It has been found experimentally that when a body is stressed within elastic limit, the lateral strain bears a constant ratio to the linear strain.

IES-18. Ans. (d)

IES-19. Ans. (b) Total load $(P) = 8 \times \sigma \times \dfrac{\pi d^2}{4}$ or $d = \sqrt{\dfrac{P}{2\pi\sigma}} = \sqrt{\dfrac{980175}{2\pi \times 315}} = 22.25\,\text{mm}$

$\therefore R_B = \dfrac{10}{3}$

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IES-22. Ans. (b)

IES-23. Ans. (d) If we resist to expand then only stress will develop.

IES-24. Ans. (d)

IES-25. Ans. (d) $\alpha E \Delta t = (12 \times 10^{-6}) \times (200 \times 10^{3}) \times (120 - 20) = 240\,MPa$

It will be compressive as elongation restricted.

IES-28. Ans.(c)
IES-30. Ans.(c)

IES-31. Ans. (b)

IES-34. Ans. (c) Shear strength of cast iron is half of its compressive strength
IES-35. Ans. (d)
IES-36. Ans. (d)
IES-37. Ans. (c)
IES-38. Ans. (c)
IES-39. Ans. (a)

$$\frac{P}{200} \times 1000 + \frac{P}{100} \times 1000 = 2 \times 10^{5} \times 0.75$$

$$P = 10000N = 10KN$$

IES-40. Ans. (d)
IES-41. Ans. (d)

$$\propto \Delta TL - 2 = \frac{\delta thermal.L}{E}$$

$$10 \times 10^{-6} \times 40 \times 20 - 2 \times 10^{-3} = \frac{\delta thermal \times 20}{200 \times 10^{9}}$$

$$\delta therm = 60MPa(Compressive)$$

IES-42. Ans. (b)

IES-43. Ans. (a)
IES-44. Ans. (b)
IES-45. Ans. (b)

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Previous Conventional Questions with Answers

Conventional Question IES-2008

Question: **What different stresses set-up in a bolt due to initial tightening, while used as a fastener? Name all the stresses in detail.**

Answer: (i) When the nut is initially tightened there will be some elongation in the bolt so tensile stress will develop.

(ii) While it is tightening a torque across some shear stress. But when tightening will be completed there should be no shear stress.

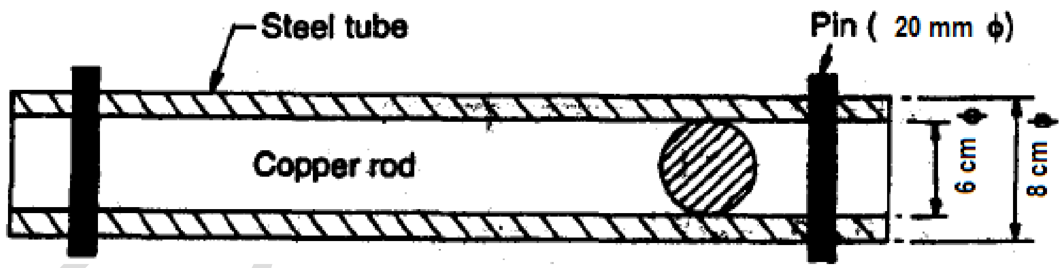
Conventional Question IES-2008

Question: **A Copper rod 6 cm in diameter is placed within a steel tube, 8 cm external diameter and 6 cm internal diameter, of exactly the same length. The two pieces are rigidly fixed together by two transverse pins 20 mm in diameter, one at each end passing through both rod and the tube.**

Calculated the stresses induced in the copper rod, steel tube and the pins if the temperature of the combination is raised by 50°C.

[Take E_S=210 GPa, $\alpha_s = 0.0000115/^oC$; E_c=105 GPa, $\alpha_c = 0.000017/^oC$]

Answer:



$$\frac{\sigma_c}{E_c} + \frac{\sigma_s}{E_s} = \Delta t(\alpha_c - \alpha_s)$$

$$\text{Area of copper rod}(A_c) = \frac{\pi d^2}{4} = \frac{\pi}{4}\left(\frac{6}{100}\right)^2 m^2 = 2.8274 \times 10^{-3} m^2$$

$$\text{Area of steel tube }(A_s) = \frac{\pi d^2}{4} = \frac{\pi}{4}\left[\left(\frac{8}{100}\right)^2 - \left(\frac{6}{100}\right)^2\right] m^2 = 2.1991 \times 10^{-3} m^2$$

Rise in temperature, $\Delta t = 50^oC$

Free expansion of copper bar $= \alpha_c L \Delta t$

Free expansion of steel tube $= \alpha_s L \Delta t$

Difference in free expansion $= (\alpha_c - \alpha_s) L \Delta t$

$$=(17-11.5)\times 10^{-6}\times L\times 50=2.75\times 10^{-4}L\,m$$

A compressive force (P) exerted by the steel tube on the copper rod opposed the extra expansion of the copper rod and the copper rod exerts an equal tensile force P to pull the steel tube. In this combined effect reduction in copper rod and increase in length of steel tube equalize the difference in free expansions of the combined system.
Reduction in the length of copper rod due to force P Newton=

$$(\Delta L)_C=\frac{PL}{A_cE_c}=\frac{PL}{(2.8275\times 10^{-3})(105\times 10^{9})}\text{m}$$

Increase in length of steel tube due to force P

$$(\Delta L)_S=\frac{PL}{A_sE_s}=\frac{P.L}{(2.1991\times 10^{-3})(210\times 10^{9})}m$$

Difference in length is equated

$$(\Delta L)_c+(\Delta L)_s=2.75\times 10^{-4}L$$

$$\frac{PL}{(2.8275\times 10^{-3})(105\times 10^{9})}+\frac{P.L}{(2.1991\times 10^{-3})(210\times 10^{9})}=2.75\times 10^{-4}L$$

Or P = 49.695 kN

$$\text{Stress in copper rod, }\sigma_c=\frac{P}{A_c}=\frac{49695}{2.8275\times 10^{-3}}\text{MPa}=17.58\text{MPa}$$

$$\textit{Stress}\text{ in steel tube, }\sigma_s=\frac{P}{A_s}=\frac{49695}{2.1991\times 10^{-3}}\text{MPa}=22.6\text{MPa}$$

Since each of the pin is in double shear, shear stress in pins (τ_{pin})

$$=\frac{P}{2\times A_{pin}}=\frac{49695}{2\times\frac{\pi}{4}(0.02)^2}=79\,\text{MPa}$$

Conventional Question IES-2007

***Question*: Explain the following in brief:**

(i) Effect of size on the tensile strength

(ii) Effect of surface finish on endurance limit.

***Answer*:** (i) When size of the specimen increases tensile strength decrease. It is due to the reason that if size increases there should be more change of defects (voids) into the material which reduces the strength appreciably.

(ii) If the surface finish is poor, the endurance strength is reduced because of scratches present in the specimen. From the scratch crack propagation will start.

Conventional Question IES-2004

Question: **Mention the relationship between three elastic constants i.e. elastic modulus (E), rigidity modulus (G), and bulk modulus (K) for any Elastic material. How is the Poisson's ratio (μ) related to these modulli?**

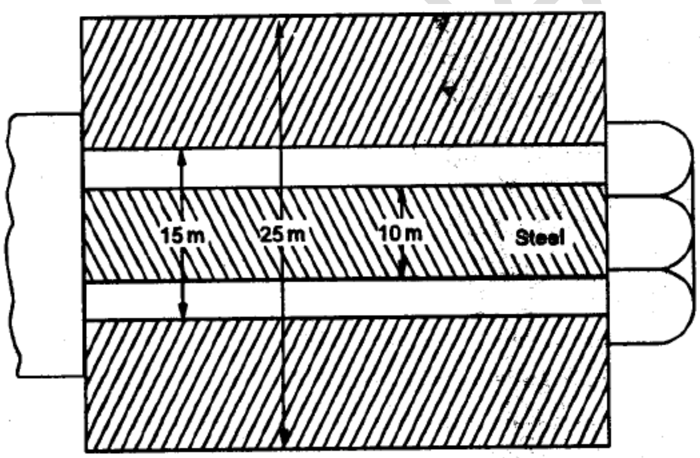
Answer: $E = \frac{9KG}{3K+G}$

$$E = 3K(1-2\mu) = 2G(1+\mu) = \frac{9KG}{3K+G}$$

Conventional Question IES-2003

Question: **A steel bolt of diameter 10 mm passes through a brass tube of internal diameter 15 mm and external diameter 25 mm. The bolt is tightened by a nut so that the length of tube is reduced by 1.5 mm. If the temperature of the assembly is raised by 40°C, estimate the axial stresses the bolt and the tube before and after heating. Material properties for steel and brass are:**
$E_S = 2\times10^5\ N/mm^2$ $\alpha_S = 1.2\times10^{-5}\ /^oC$ and $E_b = 1\times10^5\ N/mm^2$ $\alpha_b = 1.9\times10^{-15}/°C$

Answer:



Area of steel bolt $(A_s) = \frac{\pi}{4}\times(0.010)^2 m^2 = 7.854\times10^{-5} m^2$

Area of brass tube $(A_b) = \frac{\pi}{4}\left[(0.025)^2 - (0.015)^2\right] = 3.1416\times10^{-4}$

Stress due to tightening of the nut

Compressive force on brass tube = tensile fore on steel bolt

or, $\sigma_b A_b = \sigma_S A_s$

or, $E_{b.}\frac{(\Delta l)_b}{\ell}.A_b = \sigma_s A_s$ $\left[\because E = \frac{\sigma}{\in} = \frac{\sigma}{\left(\frac{\Delta L}{L}\right)}\right]$

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Let assume total length $(\ell) = 1\text{m}$

Therefore $(1\times10^5\times10^6)\times\frac{(1.5\times10^{-3})}{1}\times\left(3.1416\times10^{-4}\right) = \sigma_s\times7.854\times10^{-5}$

or $\sigma_s = 600\,\text{MPa}$ (tensile)

and $\sigma_b = E_b.\frac{(\Delta l)_b}{\ell} = (1\times10^5)\times\frac{(1.5\times10^{-3})}{1}\text{ MPa} = 150\text{ MPa}$ (Compressive)

So before heating

Stress in brass tube $(\sigma_b) = 150\text{ MPa}$ (compressive)

Stress in steel bolt$(\sigma_s) = 600\text{ MPa}$ (tensile)

Stress due to rise of temperature

Let stress $\sigma_b^{'}$ & $\sigma_s^{'}$ are due to brass tube and steel bolt.

If the two members had been free to expand,

Free expansion of steel = $\alpha_s\times\Delta t\times1$

Free expansion of brass tube = $\alpha_b\times\Delta t\times1$

Since $\alpha_b > \sigma_s$ free expansion of copper is greater than the free expansion of steel. But they are rigidly fixed so final expansion of each members will be same. Let us assume this final expansion is 'δ', The free expansion of brass tube is grater than δ, while the free expansion of steel is less than δ. Hence the steel rod will be subjected to a tensile stress while the brass tube will be subjected to a compressive stress.

For the equilibrium of the whole system,

Total tension (Pull) in steel = Total compression (Push) in brass tube.

$\sigma_b^{'}A_b = \sigma_s^{'}A_s\ or,\ \sigma_b^{'} = \sigma_s^{'}\times\frac{A_s}{A_b} = \frac{7.854\times10^{-5}}{3.14\times10^{-4}}\sigma_S^{'} = 0.25\sigma_S^{'}$

Final expansion of steel = final expansion of brass tube

$\alpha_s(\Delta t).1 + \frac{\sigma_s^{'}}{E_s}\times1 = \alpha_b(\Delta t)\times1 - \frac{\sigma_b^{'}}{E_b}\times1$

$or,\left(1.2\times10^{-5}\right)\times40\times1 + \frac{\sigma_s^{'}}{2\times10^5\times10^6} = (1.9\times10^{-5})\times40\times1 - \frac{\sigma_b^{'}}{1\times10^5\times10^6} --(ii)$

From(i) & (ii) we get

$\sigma_s^{'}\left[\frac{1}{2\times10^{11}} + \frac{0.25}{10^{11}}\right] = 2.8\times10^{-4}$

or,$\sigma_s^{'} = 37.33$ MPa (Tensile stress)

or, $\sigma_b^{'} = 9.33\,\text{MPa}$ (compressive)

Therefore, the final stresses due to tightening and temperature rise

Stress in brass tube $= \sigma_b + \sigma_b^{'} = 150 + 9.33\,\text{MPa} = 159.33\,\text{MPa}$

Stress in steel bolt $= \sigma_s + \sigma_s^{'} = 600 + 37.33 = 637.33\,\text{MPa}$.

Conventional Question IES-2004

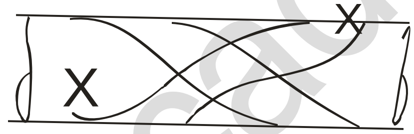
Question: **Which one of the three shafts listed hare has the highest ultimate tensile strength? Which is the approximate carbon content in each steel?**
(i) Mild Steel (ii) cast iron (iii) spring steel

Answer: Among three steel given, spring steel has the highest ultimate tensile strength.
Approximate carbon content in
(i) Mild steel is (0.3% to 0.8%)
(ii) Cost iron (2% to 4%)
(iii) Spring steel (0.4% to 1.1%)

Conventional Question IES-2003

Question: **If a rod of brittle material is subjected to pure torsion, show with help of a sketch, the plane along which it will fail and state the reason for its failure.**

Answer: Brittle materials fail in tension. In a torsion test the maximum tensile test Occurs at 45° to the axis of the shaft. So failure will occurs along a 45^{o} to the axis of the shaft. So failure will occurs along a 45° helix



So failures will occurs according to 45° plane.

Conventional Question IES-2014

Question: **A steel tube 2.5 cm external dia & 1.8 cm internal dia encloses a copper rod 16cm dia to which it is rigidly joind at each end. If at a temp. of** $20°C$ t $20°C$ **there is no longitudral stress calculate the stresses in the rod and tube when the temp. is raised to** $210°C$

$$ES = 210\,GPa \quad \& \propto s = 12\times10^{-6}\,/\,°C$$
$$EC = 100\,GPa \quad \propto c = 20\times10^{-6}\,/\,°C$$

Answer:

$$\delta st = \delta cu$$

$$\alpha\Delta TL + \frac{RL}{ASES} = \alpha cu\Delta TL - \frac{RL}{AcuEcu}$$

$$12\times10^{-6}\times(210-20) + \frac{R}{\frac{\pi}{4}\left(2.5^2 - 1.8^2\right)\times10^{-4}\times100\times10^{9}}$$

$$= 20\times10^{-6}\times(210-20) + \frac{R}{\frac{\pi}{4}(1.6)^2\times10^{-4}\times210\times10^{9}}$$

$$= 816N$$

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$$\delta s = \frac{R}{As} = \frac{816}{\frac{\pi}{4}\left(2.5^2 - 1.8^2\right)\times 10^{-4}} = 3.453\text{MPa}$$

$$\delta cu = \frac{R}{Ac} = \frac{816}{\frac{\pi}{4}\left(1.6^2\right)\times 10^{-4}} = 4.06\text{MPa}$$

Students Notes